NumericAl BoUndaries and COupling
Toulouse, January 31st - February 1st 2018

Schedule

Wednesday January 31st

12h30 - 14h00	Lunch at l'esplanade
14h00 - 15h00	Jan Nordström (Linköping University) *Numerical Solution of Initial Boundary Value Problems: Theory and Practise.* I
15h00 - 16h00	David Sanchez (INSA Toulouse)
16h00 - 16h30	coffee break
16h30 - 17h30	Joackim Bernier (ENS Rennes) *Optimality and resonances in a class of compact finite difference schemes of high order*
19h30	Dinner.

Thursday February 1st

9h00 - 10h00	Jan Nordström (Linköping University) *Numerical Solution of Initial Boundary Value Problems: Theory and Practise.* II
10h00 - 10h30	break
10h30 - 11h30	Maria Kazakova (INSA Toulouse) *Modelling shoaling and breaking waves on a mild sloping beach*
11h30 - 12h30	Corentin Audiard (Université Pierre et Marie Curie) *Traveling waves for the Euler-Korteweg system*
12h30 - 14h00	Lunch at l'esplanade
14h00 - 15h00	Jan Nordström (Linköping University) *Numerical Solution of Initial Boundary Value Problems: Theory and Practise.* III

Abstracts

C. Audiard : *Traveling waves for the Euler-Korteweg system*

The Euler-Korteweg system is a perturbation of the usual compressible Euler equations, adding capillary effects that appear as a third order, dispersive term. The global existence of dispersive solutions for small data in dimension at least 3 was proved recently, for such solutions the linear dynamic is dominant. The aim of this talk is to discuss the opposite issue, namely the existence of small traveling waves in dimension 2, which is a purely nonlinear behavior.

J. Bernier : *Optimality and resonances in a class of compact finite difference schemes of high order*

We will revisit the old problem of compact finite difference approximations of the homogeneous Dirichlet problem in dimension 1. We design a large and natural set of schemes of arbitrary high order, and we equip this set with an algebraic structure. We give some general criteria of convergence and we apply them to obtain two new results. On the one hand, we use Padé approximant theory to construct, for each given order of consistency, the most efficient schemes and we prove their convergence. On the other hand, we use diophantine approximation theory to prove that almost all of these schemes are convergent at the same rate as the consistency order, up to some logarithmic correction.

M. Kazakova : *Modelling shoaling and breaking waves on a mild sloping beach*

The mathematical modelling of coastal wave is a quite challenging issue since it is difficult to describe in the same model the dispersive effects in the shoaling zone and the energy dissipation of breakers in the surf zone. In this study we propose a new model for waves propagation over a mild slopping topography. Shearing and turbulence effects in breaking waves are taken into account by a third variable called enstrophy. With Teshukov's hypothesis of weakly sheared flows, the system is closed and features depth averaged balance equations for mass, momentum and energy. In the absence of enstrophy, the system reduces to the equations of Green-Naghdi. Enstrophy production is handled with a turbulent viscosity hypothesis and enstrophy dissipation is governed by an empirical law. Since the model is dispersive and not hyperbolic, the enstrophy equation can replace conveniently the energy equation for the numerical resolution. The equations were numerically solved with the strategy of Le Métayer et al. (2010). The scheme is rewritten for the new variables and allows us to use a hybrid method which consist in the resolution of a hyperbolic system by a Godunov type method and an elliptic equation. The non-dissipative part of the model possesses a solitary wave solution which is confirmed numerically. The numerical simulations were successfully compared to the experimental data of Hsiao et al. (2008).

J. Nordström : *Numerical Solution of Initial Boundary Value Problems: Theory and Practise*

We will go through the theoretical concepts necessary for the numerical solution of initial boundary value problems (IBVPs).

- Lecture 1 deals with well-posedness, stability and their intimate relation.
- Lecture 2 presents the concepts: summation-by-parts (SBP) operators and weak boundary conditions using simultaneous approximation terms (SATs). We will show how the SBP-SAT technique produces a stable scheme, if the IBVP is well posed. This includes a discussion on how numerical boundary conditions can be implemented in a systematically stable way.
- Lecture 3 provides a ?roadmap for how to construct a stable scheme?. We will also discuss application of the SBP-SAT technique to finite difference multi-block schemes, unstructured finite volume schemes and discontinuous Galerkin schemes. If time permits, we will discuss nonlinear problems such as the incompressible Navier-Stokes equations.

D. Sanchez : *Discrete transparent boundary conditions for KdV-BBM equations*